## I B.Tech II Semester Supplementary Examinations, Feb. 2015 MATHEMATICAL METHODS

( Common to Mechanical Engineering, Electronics \& Communication Engineering, Chemical Engineering, Bio-Medical Engineering, Information Technology, Electronics \& Computer Engineering, Mining and Petroliem Technology)
Time: 3 hours
Max Marks: 75

## Answer any FIVE Questions

All Questions carry equal marks

1. (a) Find rank of matrix using Normal form $A=\left[\begin{array}{cccc}8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4\end{array}\right]$
(b) Are the following equations consistent, if so solve them $2 \mathrm{x}+3 \mathrm{y}-\mathrm{z}-2 \mathrm{w}=2,4 \mathrm{x}+5 \mathrm{y}+3 \mathrm{z}=7$ $x+y+2 z+w=5$
2. Find Eigen Vectors of $A=\left[\begin{array}{lll}1 & 0 & 1 \\ 1 & 4 & 3 \\ 0 & 2 & 0\end{array}\right]$
3. Find the rank, signature and index of the quadratic form
$2 x_{1}^{2}+x_{2}^{2}-3 x_{3}^{2}+12 x_{1} x_{2}-4 x_{1} x_{3}-8 x_{2} x_{3}$ by reducing it to normal form .Also write the linear transformation which brings about the normal reduction
4. (a) Compute the real root of the equation $3 x=\cos x+1$ by Bisection method
(b) Compute the real root of the equation $\tan x=x$ by iteration method. [8+7]
5. (a) If the interval of differencing is unity, prove the following:
(i) $\Delta\left(\frac{2^{x}}{x!}\right)=\frac{2^{x}(1-x)}{(x+1)!}$
(ii) $\Delta\left\{\tan ^{-1}\left(\frac{n-1}{n}\right)\right\}=\tan ^{-1}\left(\frac{1}{2 n^{2}}\right)$
(b) Using the Newton's forward differences formula, find the interpolating polynomial for the function $y=f(x)$ given by $f(0)=1, f(1)=2, f(2)=1$, $f(3)=10$. Hence evaluate $f(0.75)$ and $f(-0.5)$.
6. (a) Find $\frac{d y}{d x}$ at $\mathrm{x}=7.47$ from the following table.

| X | 7.47 | 7.48 | 7.49 | 7.5 | 7.51 | 7.52 | 7.53 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 0.193 | 0.195 | 0.198 | 0.201 | 0.203 | 0.206 | 0.208 |

(b) The following Table gives the temperature $\theta$ (in degrees of Celsius) of a cooling body at different instants of time $t$ (in seconds):

| $\mathrm{t}:$ | 1 | 3 | 5 | 7 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\theta:$ | 85.3 | 74.5 | 67.0 | 60.5 | 54.3 |

Find approximately the rate of cooling at $\mathrm{t}=8$ seconds.
7. (a) Solve $\frac{d y}{d x}=\frac{1}{y+x} y(0)=1$ by R-K method and hence find $y(0.1)$
(b) Solve $\mathrm{y}^{1}=1+\mathrm{xy}$ subject to the condition $\mathrm{y}(0)=2$ by modified Euler's method and hence find $\mathrm{y}(0.1), \mathrm{y}(0.2)$

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[8+7]
$$

8. (a) Fit a curve of the type $\mathrm{y}=\mathrm{ae}^{b x}$ to the data by the method of least squares

| x | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 2.6 | 3.3 | 4.2 | 5.4 | 6.9 |

(b) .Fit straight line to the data by the method of least squares

| x | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 144 | 172.8 | 207.4 | 248.8 | 298.5 |

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1. (a) Find rank of a Matrix using Echelon form where $A=\left[\begin{array}{cccc}1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 5 & 3 & 14 & 4\end{array}\right]$
(b) Show that equations $x+y+z=6, x+2 y+3 z=14, x+4 y+7 z=30$ are consistent and solve them
2. Find Eigen vectors of $A=\left[\begin{array}{lll}2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2\end{array}\right]$
3. Determine the nature, index rank, and signature of the quadratic form
$5 x_{1}^{2}+26 x_{2}^{2}+10 x_{3}^{2}+6 x_{1} x_{2}+14 x_{1} x_{3}+4 x_{2} x_{3}$
4. (a) Find the root of the equation $x^{3}-6 x+4=0$ by Newton-Raphson's Method correct to five decimal places
(b) Find a root of the equation $x \log _{10} x=1.2$ by Bisection method.
5. (a) The values of annuities for certain ages are given for the following ages. Find the annuity at age $27 \frac{1}{2}$ using Gauss's forward interpolation formula

| Age: | 25 | 26 | 27 | 28 | 29 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Annuity: | 16.195 | 15.919 | 15.630 | 15.326 | 15.006 |

(b) Find $\mathrm{f}(2.5)$ using Newton's forward formula from the following table

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 0 | 1 | 16 | 81 | 256 | 625 | 1296 |

6. (a) The velocity v of a particle moving in a straight line covers at distance x in time t . They are related as given in the following table. Find $f^{\prime}(15)$

| X | 0 | 10 | 20 | 30 | 40 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 45 | 60 | 65 | 54 | 42 |

(b) Evaluate $\int_{0}^{1} x^{3} \mathrm{dx}$ with five sub-intervals by Trapezoidal rule. $[8+7]$
7. (a) Solve $y^{1}=3 x+y / 2, y(0)=1$ by Taylor series method and hence find $y(0.1)$, $\mathrm{y}(0.2)$
(b) Solve the equation $\frac{d y}{d x}=x y+1, \mathrm{y}(0)=1$ by Picard's method and hence find $\mathrm{y}(0.1)$
8. (a) Fit a least square parabola $\mathrm{y}=\mathrm{a}+\mathrm{bx}+\mathrm{cx}^{2}$ to the following data

| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 3 | 3 | 5 | 9 | 15 | 23 | 33 |

(b) Fit a straight line of the form $\mathrm{y}=\mathrm{a}+\mathrm{bx}$ to the following data

| x | 1 | 2 | 3 | 4 | 6 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 2.4 | 3.1 | 3.5 | 4.2 | 5 | 6 |

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1. (a) Using Echelon form, find rank of $A=\left[\begin{array}{cccc}1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & 8\end{array}\right]$
(b) Solve system of equations $\mathrm{x}+\mathrm{y}+\mathrm{z}=3,2 \mathrm{x}+3 \mathrm{y}+2 \mathrm{z}=7,4 \mathrm{x}+2 \mathrm{y}+3 \mathrm{z}=9$, using Gauss elimination method.
2. (a) Find Eigen Values and Eigen Vectors of $\left[\begin{array}{ccc}1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3\end{array}\right]$
(b) Prove that the Eigen values of a square matrix A and its transpose are same
3. Reduce the quadratic form $7 x^{2}+6 y^{2}+5 z^{2}-4 x y-4 y z$ to canonical from by diagonlzation. Also find the nature, index and signature and the linear transformation.
4. (a) Using Newton-Raphson's method find a root of $2 x-3 \sin x=5$ which is nearer to 3
(b) Find the root between 2 and 3 of the equation $x^{4}-x^{3}-2 x^{2}-6 x-4=0$ using bisection method.
$[8+7]$
5. (a) Interpolate by means of Gauss's backward formula the sales of a concern for the year 1976 for the given data

| Year: | 1940 | 1950 | 1960 | 1970 | 1980 | 1990 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Sales (in lakhs of Rs.) | 17 | 20 | 27 | 32 | 36 | 38 |

(b) Calculate f (1.30) from the following table.

| $\mathrm{X}:$ | 0.0 | 1.2 | 2.4 | 3.7 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{~F}(\mathrm{x}):$ | 3.41 | 2.68 | 1.37 | -1.18 |

6. (a) The population of a certain town (as obtained from census data) is shown in the following table:

| Year | 1951 | 1961 | 1971 | 1981 | 1991 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Population(in <br> thousand) | 19.96 | 39.65 | 58.81 | 77.21 | 94.61 |

Estimate the rate of growth of the population in the year 1981
(b) The following table gives the value of $\mathrm{f}(\mathrm{x})$ at equal intervals of x .

| x | 0 | 0.5 | 1.0 | 1.5 | 2.0 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 0.399 | 0.352 | 0.242 | 0.129 | 0.054 |
|  |  |  |  |  |  |

Evaluate $\int_{0}^{2} f(x) d x$ using Simpsons $1 / 3$ and Simpsons $3 / 8$ rule.
7. (a) Solve $\frac{d y}{d x}=\frac{1}{y+x} y(0)=1$ by R-K method and hence find $y(0.1)$
(b) Solve $y^{1}=1+x y$ subject to the condition $y(0)=2$ by modified Euler's method and hence find $\mathrm{y}(0.1), \mathrm{y}(0.2)$
8. (a) Fit a least square straight line to the following data

| x | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 16 | 19 | 23 | 26 | 30 |

(b) Fit a least square parabola $\mathrm{y}=\mathrm{a}+\mathrm{bx}+\mathrm{cx}^{2}$ to the following data

| x | 0 | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| y | 2.1 | 3.5 | 5.4 | 7.3 | 8.2 |

## Set No. 4

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1. (a) Find rank of $A=\left[\begin{array}{cccc}-1 & 2 & 1 & 8 \\ 2 & 1 & -1 & 0 \\ 3 & 2 & 1 & 7\end{array}\right]$ by using Echelon form
(b) Find rank of $A=\left[\begin{array}{cccc}1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10\end{array}\right]$ using Normal Form $\quad[7+8]$
2. Verify Cayley - Hamilton theorem and find $\mathrm{A}^{-1}$ if $A=\left[\begin{array}{ccc}1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1\end{array}\right]$
3. (a) Define quadratic form, rank, signature and index of the quadratic form.
(b) Explain about the Nature of the quadratic form and find the nature of Quadratic form $2 x_{1} x_{2}-4 x_{1} x_{3}-4 x_{2} x_{3}$
4. (a) Find a real root the equation $1+\tan ^{-1}(x)-x=0$ near $\mathrm{x}=1$ correct up to 4 decimal places using iteration method
(b) By using bisection method find an approximate root of the equation $\sin x=\frac{1}{x}$ that lies between $\mathrm{x}=1$ and $\mathrm{x}=1.5$ (measured in radians).Carryout computation upto $7^{\text {th }}$ stage.
5. (a) (i) Solve $\Delta\left(e^{a x} \log b x\right)$ (ii) Prove that $\nabla^{6} y_{8}=\Delta^{6} y_{2}$.
(b) From the following table for find $\mathrm{f}(3.3)$ using gauss forward interpolation formula.

| x | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Y}=\mathrm{f}(\mathrm{x})$ | 15.30 | 15.10 | 15.00 | 14.50 | 14.00 |

6. (a) The population of a certain town (as obtained from census data) is shown in the following table:

| Year | 1891 | 1901 | 1911 | 1921 | 1931 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Population(in <br> thousand) | 46 | 66 | 81 | 93 | 101 |

Estimate the rate of growth of the population in the year 1921
(b) When a train is moving at $30 \mathrm{~m} / \mathrm{sec}$, steam is shut off and brakes are applied. The speed of the train per second after $t$ seconds is given by

| Time (t): | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Speed(v): | 30 | 24 | 19.5 | 16 | 13.6 | 11.7 | 10 | 8.5 | 7.0 |

Using Simpson's rule, determine the distance moved by the train in 40 seconds.
7. (a) Solve $y^{1}=x-y, y(0)=1$ by modified Euler's method and find $y(0.1), y(0.2)$
(b) Apply third order R-K method to find $\mathrm{y}(0.25)$ where $\mathrm{y}^{1}=1+\mathrm{xy}$, $\mathrm{y}(0)=1[8+7]$
8. (a) Fit a curve of the type $y=a+b x+\mathrm{cx}^{2}$ to the following data

| x | 10 | 15 | 20 | 25 | 30 | 35 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 35.3 | 32.4 | 29.2 | 26.1 | 23.2 | 20.5 |

(b) Fit a curve of the type $y=a b^{x}$ to the following data by the method of least squares

| x | 1 | 2 | 5 | 10 | 20 | 30 | 40 | 50 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| y | 98.2 | 91.7 | 81.3 | 64 | 36.4 | 32.6 | 7.1 | 11.3 |

